Section 4.1

Probability Distributions

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Combining Descriptive Methods and Probabilities

In this chapter we will construct probability distributions by presenting possible outcomes along with the relative frequencies we *expect*.

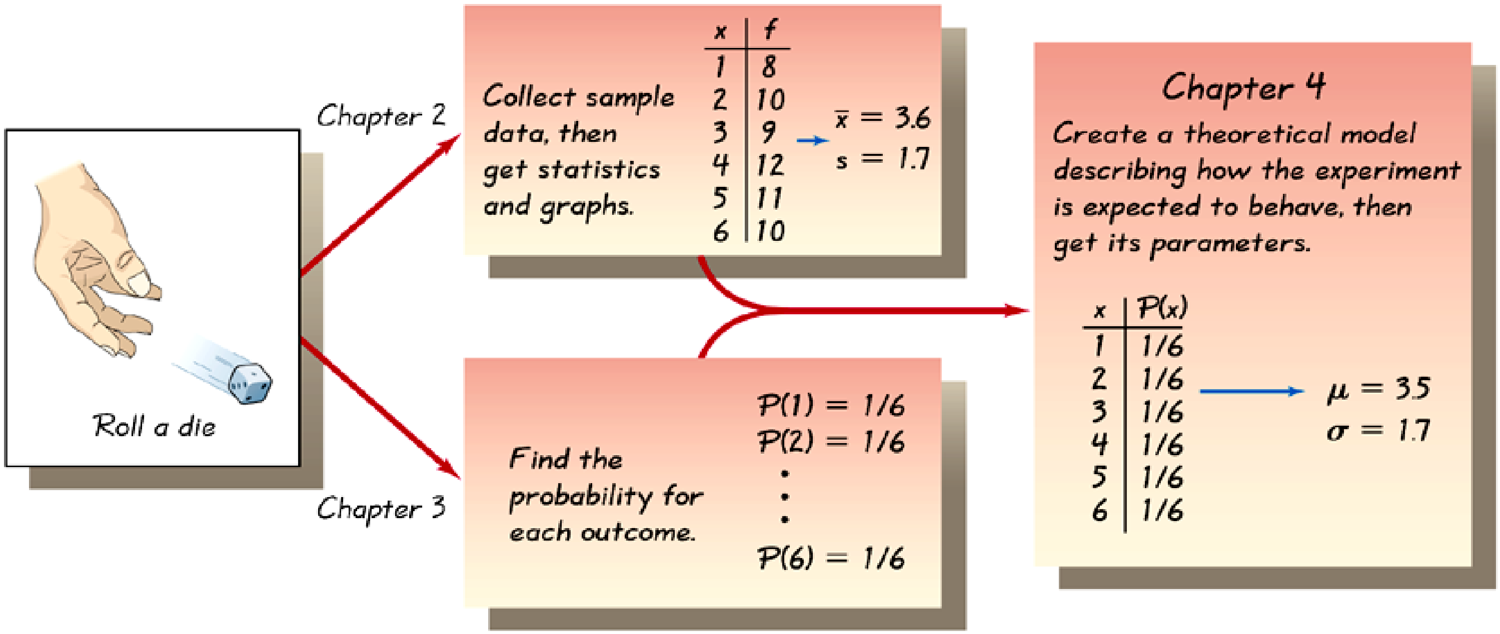

Figure 4-1

Random Variables

Random Variable

- Represents a numerical value associated with each outcome of a probability distribution.
- Denoted by ***x***
- Examples
  - *x* = Number of sales calls a salesperson makes in one day.
  - *x* = Hours spent on sales calls in one day.

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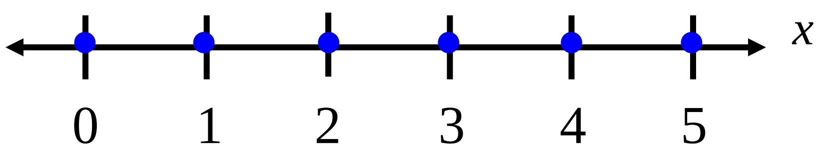
Random Variables

Discrete Random Variable

- Has a finite or countable number of possible outcomes that can be listed.
- Example
  - x = Number of sales calls a salesperson makes in one day.



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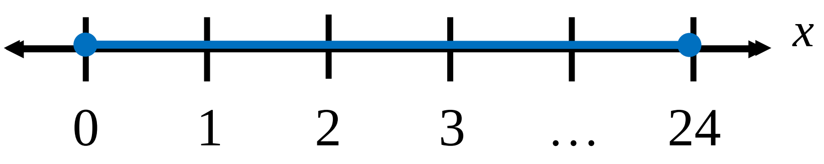
Random Variables

Continuous Random Variable

- Has an uncountable number of possible outcomes, represented by an interval on the number line.
- Example
  - x = Hours spent on sales calls in one day.



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Example: Random Variables

Decide whether the random variable *x* is discrete or continuous.

1. *x* = The number of Fortune 500 companies that lost money in the previous year.

2. *x* = The volume of gasoline in a 21-gallon tank.

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Discrete Probability Distributions

Discrete probability distribution

- Lists each possible value the random variable can assume, together with its probability.
- Must satisfy the following conditions:

In Words	*In Symbols*
1. The probability of each value of the discrete random variable is between 0 and 1, inclusive.	$0 \leq P(x) \leq 1$
2. The sum of all the probabilities is 1.	$\Sigma P(x) = 1$

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Constructing a Discrete Probability Distribution

Let x be a discrete random variable with possible outcomes $x_1, x_2, \ldots, x_n$.

1. Make a frequency distribution for the possible outcomes.
2. Find the sum of the frequencies.
3. Find the probability of each possible outcome by dividing its frequency by the sum of the frequencies.
4. Check that each probability is between 0 and 1 and that the sum is 1.

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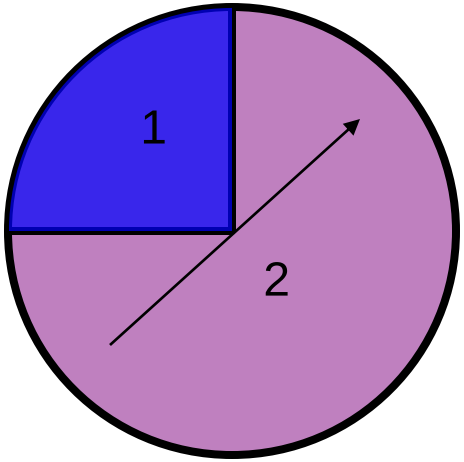
Constructing a Discrete Probability Distribution

Example:

The spinner below is divided into two sections. The probability of landing on the 1 is 0.25. The probability of landing on the 2 is 0.75. Let *x* be the number the spinner lands on. Construct a probability distribution for the random variable *x*.



x	$P(x)$
1	0.25
2	0.75

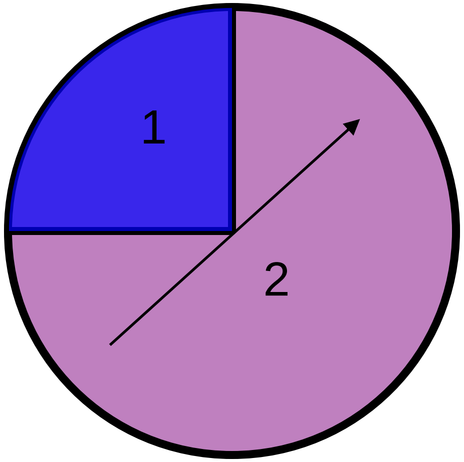
Each probability is between 0 and 1.

The sum of the probabilities is 1.

Constructing a Discrete Probability Distribution

Example:

The spinner below is spun two times. The probability of landing on the 1 is 0.25. The probability of landing on the 2 is 0.75. Let *x* be the sum of the two spins. Construct a probability distribution for the random variable *x*.



The possible sums are 2, 3, and 4.

P (sum of 2) = 0.25 × 0.25 = 0.0625

Spin a 1 on the first spin.

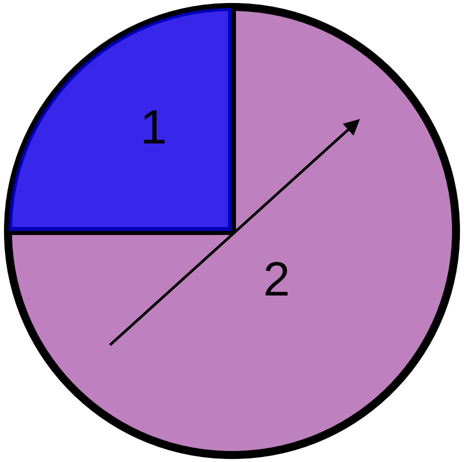
"and"

Spin a 1 on the second spin.

Continued.

Constructing a Discrete Probability Distribution

Example continued:



P (sum of 3) = 0.25 $\times$ 0.75 = 0.1875

Spin a 1 on the first spin. | "and" | Spin a 2 on the second spin.

"or"

P (sum of 3) = 0.75 $\times$ 0.25 = 0.1875

Spin a 2 on the first spin. | "and" | Spin a 1 on the second spin.

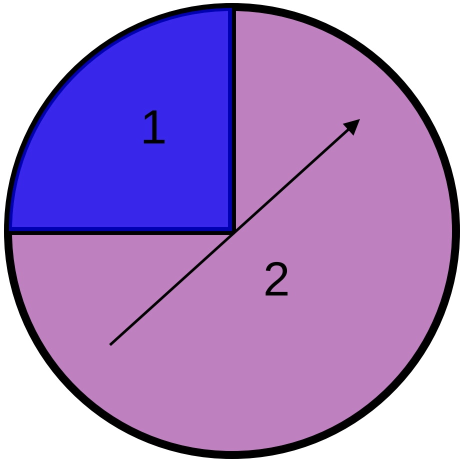
Sum of spins, x	$P(x)$
2	0.0625
3	0.375
4	

0.1875 + 0.1875

Continued.

Constructing a Discrete Probability Distribution

Example continued:



P (sum of 4) = 0.75 × 0.75 = 0.5625

Spin a 2 on the first spin.

"and"

Spin a 2 on the second spin.

Sum of spins, x	$P(x)$
2	0.0625
3	0.375
4	0.5625

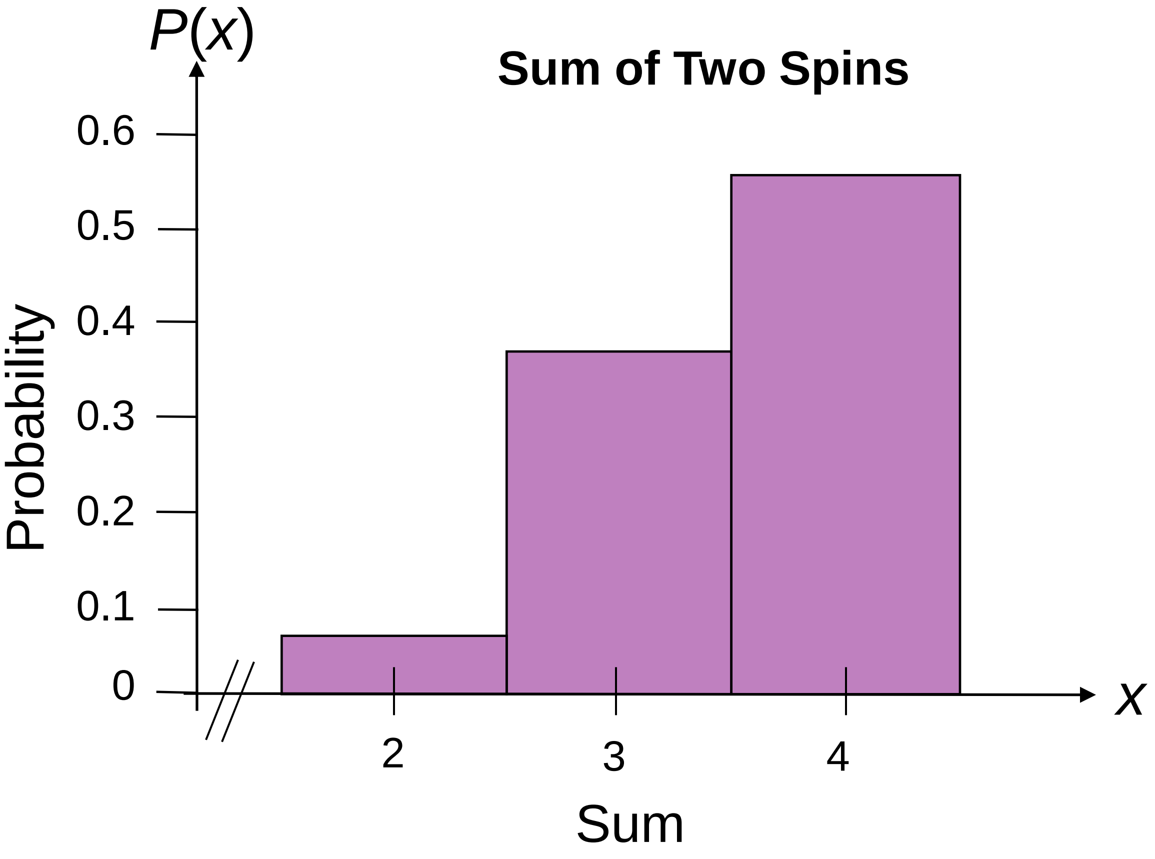
Each probability is between 0 and 1, and the sum of the probabilities is 1.

Graphing a Discrete Probability Distribution

Example:

Graph the following probability distribution using a histogram.

Sum of spins, x	$P(x)$
2	0.0625
3	0.375
4	0.5625



Mean

The **mean** of a discrete random variable is given by

$$\mu = \Sigma xP(x).$$

Each value of *x* is multiplied by its corresponding probability and the products are added.

Example:

Find the mean of the probability distribution for the sum of the two spins.

x	$P(x)$	$xP(x)$
2	0.0625	$2(0.0625) = 0.125$
3	0.375	$3(0.375) = 1.125$
4	0.5625	$4(0.5625) = 2.25$

$\Sigma xP(x) = 3.5$

The mean for the two spins is 3.5.

Variance

The **variance** of a discrete random variable is given by

$$\sigma^2 = \Sigma(x - \mu)^2 P(x).$$

Example:

Find the variance of the probability distribution for the sum of the two spins. The mean is 3.5.

x	$P(x)$	$x - \mu$	$(x - \mu)^2$	$P(x)(x - \mu)^2$
2	0.0625	–1.5	2.25	≈ 0.141
3	0.375	–0.5	0.25	≈ 0.094
4	0.5625	0.5	0.25	≈ 0.141

$\Sigma P(x)(x - 2)^2$

≈ 0.376

The variance for the two spins is approximately 0.376

Standard Deviation

The **standard deviation** of a discrete random variable is given by

$$\sigma = \sqrt{\sigma^2}.$$

Example:

Find the standard deviation of the probability distribution for the sum of the two spins. The variance is 0.376.

x	$P(x)$	$x - \mu$	$(x - \mu)^2$	$P(x)(x - \mu)^2$
2	0.0625	–1.5	2.25	0.141
3	0.375	–0.5	0.25	0.094
4	0.5625	0.5	0.25	0.141

$$\sigma = \sqrt{\sigma^2}$$
$$= \sqrt{0.376} \approx 0.613$$

Most of the sums differ from the mean by no more than 0.6 points.

Mean

Mean of a discrete probability distribution

- $\mu = \Sigma x P(x)$
- Each value of *x* is multiplied by its corresponding probability and the products are added.

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Variance and Standard Deviation

Variance of a discrete probability distribution

- $\sigma^2 = \Sigma(x - \mu)^2 P(x)$

Standard deviation of a discrete probability distribution

- $\sigma = \sqrt{\sigma^2} = \sqrt{\Sigma(x-\mu)^2 P(x)}$

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Expected Value

Expected value of a discrete random variable

- Equal to the mean of the random variable.
- $E(x) = \mu = \Sigma x P(x)$

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Section 4.3

Binomial Distributions

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Binomial Experiments

1. The experiment is repeated for a fixed number of trials, where each trial is independent of other trials.
2. There are only two possible outcomes of interest for each trial. The outcomes can be classified as a success (*S*) or as a failure (*F*).
3. The probability of a success, $P(S)$, is the same for each trial.
4. The random variable x counts the number of successful trials.

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Notation for Binomial Experiments

Symbol	*Description*
n	The number of times a trial is repeated
$p = P(S)$	The probability of success in a single trial
$q = P(F)$	The probability of failure in a single trial ($q = 1 - p$)
x	The random variable represents a count of the number of successes in n trials: $x = 0, 1, 2, 3, \ldots, n$.

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Example: Binomial Experiments

Decide whether the experiment is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x.

1. A certain surgical procedure has an 85% chance of success. A doctor performs the procedure on eight patients. The random variable represents the number of successful surgeries.

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Solution: Binomial Experiments

Binomial Experiment

1. Each surgery represents a trial. There are eight surgeries, and each one is independent of the others.
2. There are only two possible outcomes of interest for each surgery: a success (*S*) or a failure (*F*).
3. The probability of a success, $P(S)$, is 0.85 for each surgery.
4. The random variable x counts the number of successful surgeries.

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Solution: Binomial Experiments

Binomial Experiment

- $n = 8$ (number of trials)
- $p = 0.85$ (probability of success)
- $q = 1 - p = 1 - 0.85 = 0.15$ (probability of failure)
- $x = 0, 1, 2, 3, 4, 5, 6, 7, 8$ (number of successful surgeries)

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Example: Binomial Experiments

Decide whether the experiment is a binomial experiment. If it is, specify the values of n, p, and q, and list the possible values of the random variable x.

2. A jar contains five red marbles, nine blue marbles, and six green marbles. You randomly select three marbles from the jar, *without replacement*. The random variable represents the number of red marbles.

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Solution: Binomial Experiments

Not a Binomial Experiment

- The probability of selecting a red marble on the first trial is 5/20.
- Because the marble is not replaced, the probability of success (red) for subsequent trials is no longer 5/20.
- The trials are not independent and the probability of a success is not the same for each trial.

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Example: Finding Binomial Probabilities

Microfracture knee surgery has a 75% chance of success on patients with degenerative knees. The surgery is performed on three patients. Find the probability of the surgery being successful on exactly two patients.

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Binomial Probability Distribution

Binomial Probability Distribution

- List the possible values of x with the corresponding probability of each.
- Example: Binomial probability distribution for Microfacture knee surgery: $n = 3$, $p = \frac{3}{4}$

x	0	1	2	3
$P(x)$	0.016	0.141	0.422	0.422

  - Use binomial probability formula to find probabilities.

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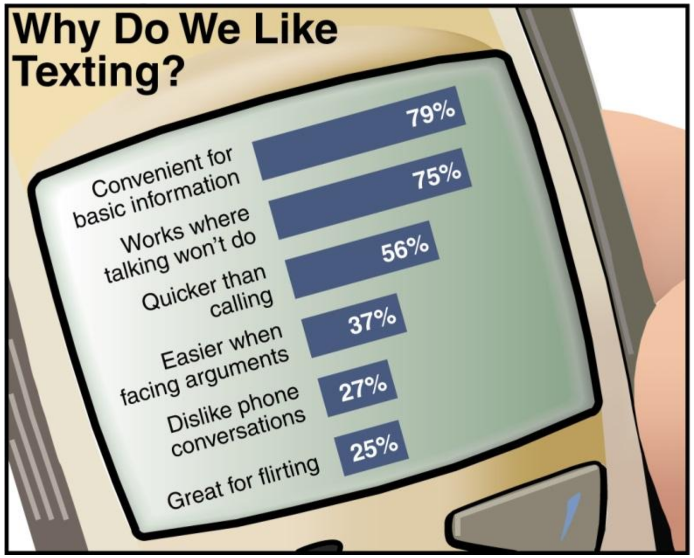
Example: Constructing a Binomial Distribution

In a survey, U.S. adults were asked to give reasons why they liked texting on their cellular phones. Seven adults who participated in the survey are randomly selected and asked whether they like texting because it is quicker than Calling. Create a binomial probability distribution for the number of adults who respond yes.



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Solution: Constructing a Binomial Distribution

x	$P(x)$
0	0.0032
1	0.0284
2	0.1086
3	0.2304
4	0.2932
5	0.2239
6	0.0950
7	0.0173

All of the probabilities are between 0 and 1 and the sum of the probabilities is $1.00001 \approx 1$.

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Example: Graphing a Binomial Distribution

Sixty percent of households in the U.S. own a video game console. You randomly select six households and ask each if they own a video game console. Construct a probability distribution for the random variable *x*. Then graph the distribution. *(Source: Deloitte, LLP)*

Solution:

- $n = 6$, $p = 0.6$, $q = 0.4$
- Find the probability for each value of *x*

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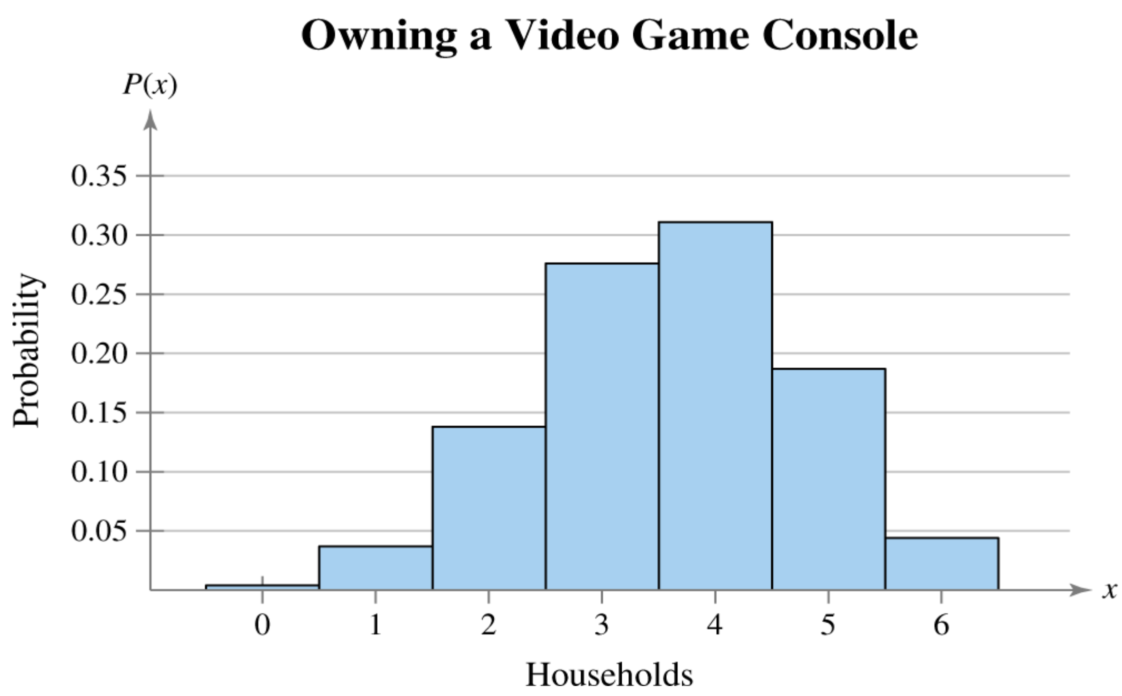
Solution: Graphing a Binomial Distribution

x	0	1	2	3	4	5	6
P(x)	0.004	0.037	0.138	0.276	0.311	0.187	0.047

Histogram:



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Mean, Variance, and Standard Deviation

- **Mean:** $\mu = np$
- **Variance:** $\sigma^2 = npq$
- **Standard Deviation:** $\sigma = \sqrt{npq}$

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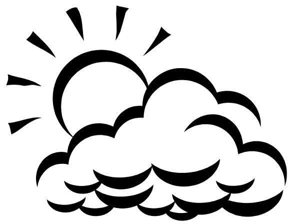
Example: Finding the Mean, Variance, and Standard Deviation

In Pittsburgh, Pennsylvania, about 56% of the days in a year are cloudy. Find the mean, variance, and standard deviation for the number of cloudy days during the month of June. Interpret the results and determine any unusual values. *(Source: National Climatic Data Center)*



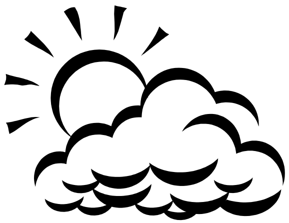
Solution: $n = 30,\ p = 0.56,\ q = 0.44$

Mean: $\mu = np = 30 \cdot 0.56 = 16.8$

Variance: $\sigma^2 = npq = 30 \cdot 0.56 \cdot 0.44 \approx 7.4$

Standard Deviation: $\sigma = \sqrt{npq} = \sqrt{30 \cdot 0.56 \cdot 0.44} \approx 2.7$

Solution: Finding the Mean, Variance, and Standard Deviation



$\mu = 16.8 \qquad \sigma^2 \approx 7.4 \qquad \sigma \approx 2.7$

- On average, there are 16.8 cloudy days during the month of June.
- The standard deviation is about 2.7 days.
- Values that are more than two standard deviations from the mean are considered unusual.
  - 16.8 – 2(2.7) =11.4; A June with 11 cloudy days or less would be unusual.
  - 16.8 + 2(2.7) = 22.2; A June with 23 cloudy days or more would also be unusual.

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